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13. ABSTRACT *(Maximum 200 words)*
The Project entailed investigating several problems in the parallel solution of sparse systems of linear equations and eigenproblems, including:
Algorithms for factoring sparse matrices; techniques for the solution of sparse triangular systems; iterative methods for sparse systems, focusing mainly on preconditioning techniques for conjugate-direction methods; the solution of the symmetric tridiagonal eigenproblem.
While this may seem to be an eclectic group of topics, there are, in fact close relationships among them. As one example, a common technique for preconditioning iterative methods depends crucially on efficient solution of triangular systems. As another, it should be possible to construct an effective Lanczos-type algorithm for sparse, symmetric eigenproblems by combining the techniques required for conjugate-direction iterations for linear systems with those required for the solution of symmetric tridiagonal eigenproblems.

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FINAL REPORT

Linear Algebraic Computation on Distributed Memory Parallel Machines
U. S. Army Research Office Contract DAAL03-91-G-0032
1 February 1991 – 30 June 1994

Principal Investigator:

Stanley C. Eisenstat, Department of Computer Science, Yale University

Statement of the Problem Studied:

The project entailed investigating several problems in the parallel solution of sparse systems of linear equations and eigenproblems, including:

- algorithms for factoring sparse matrices;
- techniques for the solution of sparse triangular systems;
- iterative methods for sparse systems, focusing mainly on preconditioning techniques for conjugate-direction methods;
- the solution of the symmetric tridiagonal eigenproblem.

While this may seem to be an eclectic group of topics, there are, in fact, close relationships among them. As one example, a common technique for preconditioning iterative methods depends crucially on efficient solution of triangular systems. As another, it should be possible to construct an effective Lanczos-type algorithm for sparse, symmetric eigenproblems by combining the techniques required for conjugate-direction iterations for linear systems with those required for the solution of symmetric tridiagonal eigenproblems.

Summary of the Most Important Results:

The main focus has been on algorithms for the symmetric tridiagonal eigenvalue problem, including the bidiagonal singular value problem and the problem of updating and downdating the singular value decomposition (SVD):

- We have devised a stable and efficient algorithm for finding the spectral decomposition of a matrix that is the sum of a diagonal matrix and a rank-one matrix. This is the key subproblem in divide-and-conquer algorithms for the symmetric tridiagonal eigenproblem (including Cuppen's method [Cuppen, Dongarra+Sorensen]), and in computing the spectral decomposition of a symmetric rank-one modification of a symmetric matrix whose eigendecomposition is known. All previous approaches required the use of extended precision arithmetic in order to maintain orthogonality of the eigenvectors [Kahan, Sorensen+Tang]. The new algorithm is based on a novel and *stable* method for computing the eigenvectors (and thus does *not* require extended precision), yet it is as efficient as previous approaches and just as parallel.
- We have devised a new arrowhead divide-and-conquer method for the symmetric tridiagonal eigenproblem; a new divide-and-conquer method for finding the SVD of a bidiagonal matrix; and new methods for updating and downdating the SVD of a dense matrix when a row or column is added. The approach above is used to stabilize each

of these algorithms. Moreover, we have shown how to update the orthogonal factor(s) more efficiently than previous algorithms do by taking full advantage of the nonzero structure without reducing parallelism.

- We have shown how to use the fast multipole method of Greengard and Rokhlin to accelerate these procedures. For example, the straight-forward implementation of the arrowhead divide-and-conquer algorithm computes all the eigenvalues of an $N \times N$ matrix in $O(N^2)$ time, and both the eigenvalues and eigenvectors in $O(N^3)$ time. The accelerated method computes all the eigenvalues in $O(N \log_2 N)$ time, and both the eigenvalues and eigenvectors in $O(N^2)$ time.

Another focus has been the problem of computing the rank-revealing QR factorization. A new, *strong* rank-revealing QR decomposition of a matrix has been defined. Unlike the original rank-revealing QR decomposition of [Chan], the new decomposition leads to stable algorithms for finding a basis for an approximate right null space of a matrix or for separating the linearly independent columns of a matrix from the linearly dependent ones. The existence of such decompositions has been proved, and an algorithm has been devised that computes them in nearly the same time as the ordinary QR decomposition.

A third focus has been the discovery of a new technique for deriving bounds on the *relative* change in the singular values of a real matrix (or the eigenvalues of a real symmetric matrix) due to a certain class of perturbations, as well as bounds on the angles between the unperturbed and perturbed singular vectors (or eigenvectors). This class of perturbations include component-wise relative perturbations of the entries in a bidiagonal or biacyclic matrix, and perturbations that annihilate the off-diagonal block in a block triangular matrix. The technique can be used to give very simple proofs of many existing relative perturbation and deflation bounds, and some new relative perturbation and deflation results for the singular values, vectors, and invariant subspaces of biacyclic, triangular, and shifted triangular matrices.

The final focus has been on taking advantage of structural symmetry in sparse elimination. We have shown how to improve the performance of a class of partial pivoting codes for the LU factorization of large sparse unsymmetric matrices. Experimental results demonstrate the effectiveness of this technique in reducing the overall factorization time. Moreover, this approach has been generalized to create a new, supernodal algorithm for this problem. The new algorithm makes heavy use of the BLAS-2 and thus runs significantly faster.

Publications:

1. Stanley C. Eisenstat and Ilse C. F. Ipsen, "Relative perturbation bounds for eigenspaces and singular vector subspaces," in John G. Lewis, Editor, *Proceedings of the Fifth SIAM Conference on Applied Linear Algebra*, SIAM, June 1994, pp. 62–66.
2. Stanley C. Eisenstat and Ilse C. F. Ipsen, "Relative perturbation techniques for singular value problems," Research Report YALEU/DCS/RR-942, Department of Computer Science, Yale University, July 1993 (to appear in *SIAM Journal on Numerical Analysis*).
3. Stanley C. Eisenstat and J. W. H. Liu, "Exploiting structural symmetry in a sparse partial pivoting code," *SIAM Journal on Scientific Computing 14*(1):253-257, January 1993.
4. Stanley C. Eisenstat and J. W. H. Liu, "Structural representations of Schur complements

in sparse matrices," in Alan George, John R. Gilbert, and Joseph W. H. Liu, Editors, *Graph Theory and Sparse Matrix Computation*, Volume 56 of *The IMA Volumes in Mathematics and its Applications*, Springer-Verlag, 1993, pp. 85–100.

5. Stanley C. Eisenstat and Homer F. Walker, "Choosing the forcing terms in an inexact Newton method" (submitted to *SIAM Journal on Scientific Computing*).
6. Ming Gu, "Studies in Numerical Linear Algebra," Ph. D. dissertation, Department of Computer Science, Yale University, July 1993.
7. Ming Gu and Stanley C. Eisenstat, "A divide-and-conquer algorithm for the bidiagonal SVD," Research Report YALEU/DCS/RR-933, Department of Computer Science, Yale University, December 1992 (to appear in *SIAM Journal on Matrix Analysis and Applications*).
8. Ming Gu and Stanley C. Eisenstat, "A divide-and-conquer algorithm for the symmetric tridiagonal eigenproblem," Research Report YALEU/DCS/RR-932, Department of Computer Science, Yale University, November 1992 (to appear in *SIAM Journal on Matrix Analysis and Applications*).
9. Ming Gu and Stanley C. Eisenstat, "Downdating the singular value decomposition," Research Report YALEU/DCS/RR-939, Department of Computer Science, Yale University, May 1993 (to appear in *SIAM Journal on Matrix Analysis and Applications*).
10. Ming Gu and Stanley C. Eisenstat, "An efficient algorithm for computing a strong rank-revealing QR factorization," Research Report YALEU/DCS/RR-967, Department of Computer Science, Yale University, May 1994 (submitted to *SIAM Journal on Scientific Computing*).
11. Ming Gu and Stanley C. Eisenstat, "Relative perturbation theory for eigenproblems," Research Report YALEU/DCS/RR-934, Department of Computer Science, Yale University, February 1993 (submitted to *SIAM Journal on Numerical Analysis*).
12. Ming Gu and Stanley C. Eisenstat, "A stable algorithm for the rank-one modification of the symmetric eigenproblem," Research Report YALEU/DCS/RR-916, Department of Computer Science, Yale University, September 1992 (to appear in *SIAM Journal on Matrix Analysis and Applications*).

Participating Scientific Personnel:

1. Stanley C. Eisenstat
2. Ming Gu (Ph. D., Department of Computer Science, Yale University, November 1993)